SOS POLITICAL SCIENCE AND PUBLIC ADMINISTRATION

MBA HRD 206

SUBJECT NAME: QUANTITATIVE TECHNIQUES FOR MANAGERS

TOPIC NAME:

Multiplication Theorem on Probability

In conditional probability, we know that the probability of occurrence of some event is affected when some of the possible events have already occurred. When we know that a particular event B has occurred, then instead of S, we concentrate on B for calculating the probability of occurrence of event A given B.

Taking the above example of throwing of two dice, the possible outcomes are

S = {(x, y): x, y = 1, 2, 3, 4, 5, 6}.

There are 36 elements in the sample space S. The probability of occurrence of any of the possible outcome is $P(E_i) = 1/36$. We don't know the result of the throw of the dice by the friend. However, we have the information that the sum of the numbers is even. Let us find how this information is affecting the probability of the outcome.

Event A shows the sum of the numbers is an even number. A = {(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2), (6, 4), (6, 6)}. This means that out of 36 outcomes these 18 outcomes are now only possible and the remaining are not.

The probability for each of these outcomes is $P(A |E_i) = 18/36 = ½$. This example shows that some additional information may change the probability of the happening of some event.

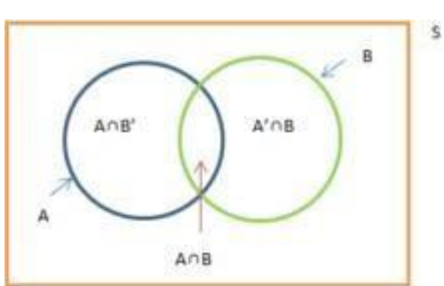
Theorem 1

For two events A and B,

$P(A \cap B) = P(A)\ P(B \mid A),\ P(A) > 0.$

or, $P(A \cap B) = P(B)\ (A \mid B),\ P(B) > 0.$

Here, $P(B \mid A)$ represents the conditional probability of occurrence of B when the event A had already occurred. $P(A \mid B)$ represents the conditional probability of occurrence of A when the event B had already happened.



Proof: From the concept of conditional probability, we have

$P(A \mid B) = P(A \cap B) / P(B)$.

Re-writing the above, we have, $P(A \cap B) = P(B)\, P(A \mid B)$.

Similary, $P(B \mid A) = P(A \cap B) / P(A)$.

$\Rightarrow P(A \cap B) = P(A)\, P(B \mid A)$.

The mathematical theorem on probability shows that the probability of the simultaneous occurrence of two events A and B is equal to the product of the probability of one of these events and the conditional probability of the other, given that the first one has occurred.

Theorem 2

For two events A and B such that $P(B) > 0$, $P(A \mid B) \leq P(A)$.

Proof: It is obvious that the number of common outcomes in A and B is either less or equal to the number of outcomes in any of the event.

$n(A \cap B) \leq n(A) \ldots$ (i),

and, $n(B) \leq n(S) \ldots$ (ii)

Dividing (i) and (ii), we get,

$n(A \cap B) / n(B) \leq n(A) / n(S)$

$\Rightarrow P(A \mid B) \leq P(A)$.

Multiplication Theorem for Independent Events

The multiplication theorem on probability for dependent events can be extended for the independent events. From the theorem, we have, $P(A \cap B) = P(A)\, P(B \mid A)$. If the events A and B are independent, then, $P(B \mid A) = P(B)$. The above theorem reduces to

$P(A \cap B) = P(A)\, P(B)$.

This shows that the probability that both of these occur simultaneously is the product of their respective probabilities.

Extension of Multiplication Theorem of Probability to n Events

For n events $A_1, A_2, \ldots, A_n$, we have

$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1)\, P(A_2 \,|\, A_1)\, P(A_3 \,|\, A_1 \cap A_2) \ldots \times P(A_n \,|A_1 \cap A_2 \cap \ldots \cap A_{n-1})$

Extension of Multiplication Theorem of Probability to n Independent Events

For n independent events, the multiplication theorem reduces to

$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1)\, P(A_2) \ldots P(A_n)$.

Solved Example for You

Question 1: A box contains 5 black, 7 red and 6 green balls. Three balls are drawn from this box one after the other without replacement. What is the probability that the three balls are

1. **all black balls**
2. **of different colors**
3. **two black and one green black.**

Answer : The total number of the balls in the box is 5 + 7 + 6 = 18. Let events

B: drawing black balls.
R: drawing red balls.
G: drawing green balls.

The balls are drawn without replacement. For the first draw, there are 18 balls to choose from. The number of balls gets lessened by 1 for the second draw i.e., 18 − 1 = 17 and 16 for the third draw.

1. Probability that the three balls are all black = $P(B_1)\, P(B_2 \,|\, B_1)\, P(B_3 \,|\, B_1 \cap B_2) = 5⁄18 \times 4⁄17 \times 3⁄16 = 5⁄408$.
2. The probability that the three balls are all different in color = $P(B_1)\, P(R_1 \,|\, B_1)\, P(G_1 \,|\, B_1 \cap R_1) = 5⁄18 \times 7⁄17 \times 6⁄16 = 35⁄816$.
3. Probability that two black and one green balls are drawn = $P(B_1)\, P(B_2 \,|\, B_1)\, P(G_1 \,|\, B_1 \cap B_2) = 5⁄18 \times 4⁄17 \times 6⁄16 = 5⁄204$.

It does not matter which color ball is drawn first.

Question 2: What is the multiplication theorem for probability?

Answer: The theorem says that 'the probability of the concurrent occurrence of 2 events that are self-determining is provided by the product of their separate probabilities.

Question 3: What is the law for multiplication?

Answer: The multiplication law states that "the probability of happening of given 2 events or in different words the probability of the intersection of 2 given events is equivalent to the product achieved by finding out the product of the probability of happening of both the events."

Question 4: What are the rules for probability?

Answer: We have 3 basic rules that associate with the probability, these are: Addition, Multiplication, and the Complement rules. We express them as: ‘$P(A \text{ or } B) = P(A) + P(B) – P(A\ B)$’.

Question 5: What are the 3 axioms of the probability?

Answer: The 3 axioms of the probability are as follows:

1. In an event A, ‘$P(A) \geq 0$’. In English, that’s ‘For an event A, the probability of ‘A’ is superior or equal to zero (0)’.
2. When ‘S’ is the sample space in an experiment i.e. the set of all possible results, ‘$P(S) = 1$’.
3. In case ‘A’ and ‘B’ are commonly exclusive products, ‘$P(A \cup B) = P(A) + P(B)$’.